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The effect of fabric coverings on the acoustic performance of modular absorbers

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Summary

Acoustic treatment consisting of a standard size of module with different internal and front panel constructions has been widely used, particularly in the BBC, for the control of the acoustic enviroment of rooms such as studios, cubicles and listening rooms. Visually, this acoustic treatment has become unacceptable and so it is common practice to cover the treatment with a lightweight stretched fabric. This report considers the effects of this fabric covering on the acoustic performance of the treatment and gives the results of some experimental measurements of these effects.

It is concluded that the fabric covering has no significant effect on the performance of acoustic treatment which absorbs most of the incident sound energy. However, at high frequencies some acoustic treatments are effective reflectors of sound energy and, under these circumstances, the effects of the fabric covering can be significant.

Although only modular absorbers are considered, the results can be applied to all types of surfaces.

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1. Introduction

In the design of studios, cubicles and other broadcasting areas, the acoustic environment must be controlled to suit the size of the rooms and the use to which the room will be put. One measure of the environment is the reverberation time, which must be controlled to lie within close limits over a range of frequencies. By providing means of sound energy absorption, the reverberation time can be reduced and, as the inherent sound energy absorption of the structure is rarely sufficient, additional means of sound energy absorption is necessary. This additional material is usually called 'acoustic treatment'.

Acoustic treatment consisting of standard size boxes with different internal constructions and different front panels has been in common use for about 12 years. The standard size simplifies the physical layout of the treatment during the initial design of the studio or cubicle and also allows the treatment to be reused after refurbishing of the area. The different internal constructions and front-panels allow a wide range of acoustic performances to be obtained for the same module size. This system has been found to be satisfactory in most respects. However, over the last few years, some dissatisfaction with the visual appearance has been expressed, both by the architects designing the areas, and the users who have to occupy the areas. To overcome this defect it has been common practice to provide a fabric covering, either in individual pieces attached to the modules or in relatively large pieces supported by an independent wooden frame in front of the treatment.

To minimise the effect of this fabric on the acoustic performance of the treatment, the fabric specified must have a very low acoustic impedance. A number of such fabrics, selected on the basis of flow-resistance measurements, have thus been approved for such applications. However, no measurements had ever been made of the effect of these coverings on the acoustic performance of the treatment. No comments about an adverse effect have been received but it was thought that such measurements should be carried out, especially as almost all new and refurbished areas are being designed with such fabric coverings over the acoustic treatment.

2. Theoretical

2.1. Fabric in free space

Fig. 1 shows the impedance analogy representing a plane sound wave, normally incident on a unit area of an infinite sheet of stretched fabric in free space. $\rho_0 c$ represents the free space acoustic impedance and is purely resistive for a plane wave. The compliance component of the fabric impedance, C_F, arises from the tension in the stretched fabric and is lower for highly tensioned conditions. Even though the mass component, M_F, is small, the resonant frequency of this system $(=1/2\pi\sqrt{M_F\ C_F})$ will be below the frequency range normally considered in room acoustics for all reasonable values of the tension in the fabric. Thus, only the real (resistive) component, R_F, is of any significance.

The method by which the proportion of the incident sound energy absorbed by the fabric can be calculated is given in Appendix 1.

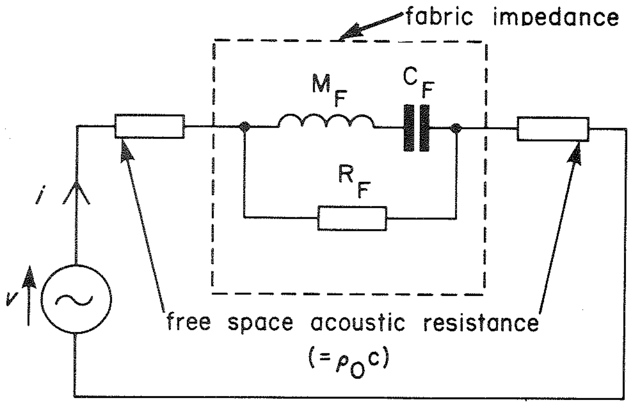

Fig. 1 – Impedance analogy of stretched fabric in free space

From the appendix, equation A1.6 can be modified to suit the case of an infinite sheet of fabric in free space, giving

$$\alpha = 4x/(2+x)^2 \quad \text{.......} \quad (1)$$

where α is the absorption coefficient, and x is the fabric resistance, normalized to the free space acoustic impedance (resistance). The maximum value of α given by equation 1 can be shown to be 0.5 and occurs when the acoustic resistance of the fabric is equal to twice the free-space radiation resistance.*

For fabric resistances typical of the approved types of furnishing fabrics ($\approx$ 40m.k.s.rayls), x is approximately 0.1 and the absorption coefficient is approximately 0.09. This value is so small as to be negligible for the purposes of ordinary acoustic design, unless the area of fabric involved is an exceptionally large fraction of the total room surface area.

*At standard temperature and pressure, $\rho_0 c$ = 407 mks rayls.

Large areas of lightweight fabric are rarely positioned in studio or cubicle areas sufficiently distant from other surfaces as to be considered as in free space. However, heavyweight fabrics are used, usually in drama studios for curtains dividing one end with a "live" acoustic from the other end with a 'dead' acoustic.* Fabrics used for this purpose are usually of high acoustic resistance, values of x equal to or greater than 2 are quite common, and the area involved is often large. The sound energy absorption can therefore be significant and must be taken into account during the acoustic design of the area.

2.2. Fabrics spaced at a finite distance from a surface.

If a stretched fabric sheet is placed parallel to and at a finite distance in front of another surface then, in general, the absorption coefficient of the combination differs from both the absorption coefficient of the uncovered surface and the absorption coefficient based on that fabric suspended in free space. Appendix 2 gives a theoretical treatment of such a combination, based on plane waves normally incident on a unit area from an infinite surface, for the case where the surface acoustic impedance is purely resistive.

The general result obtained for the absorption coefficient of the combination α' is given by (equation A2.12)

$$\alpha' = 4(Z_R + x)/((Z_R + x + 1)^2 + Z_I{}^2) \quad \text{............ (2)}$$

where $Z_R = y(\cot^2(kl) + 1)/(\cot^2(kl) + y^2)$

$Z_I = (1 - y^2)\cot(kl)/(\cot^2(kl) + y^2)$

and for
- $k = 2\pi f/c$
- x = normalised acoustic resistance of fabric
- y = normalised acoustic resistance of surface
- l = length of airspace in the direction of sound propagation
- f = frequency of sound wave
- c = velocity of sound in the medium

The normalized surface resistance, y, is only known directly for the two extreme cases, the first where the uncovered surface is a perfect absorber ($y = 1$), and the second where the uncovered surface is a perfect reflector ($y = \infty$).

In the case where the surface is a perfect absorber, equation 2 can be simplified to

$$\alpha' = 4(1 + x)/(2 + x)^2 \quad \text{.................................... (3)}$$

For all positive, non-zero values of x, α' is less than α, that is , the fabric-covered absorber will absorb less sound energy than the uncovered absorber. For example, if $\alpha = 1$ and $x = 0.1$ (a typical value for an approved lightweight fabric) then, from equation 3, $\alpha' = 0.998$. Thus, the changes brought about by the use of reasonably lightweight fabrics in the efficiency of effective acoustic treatment are not large and can safely be ignored for the purposes of acoustic design.

In the case where the surface covered by fabric is a perfect reflector, equation 2 can be simplified to

$$\alpha' = 4x/((1 + x)^2 + \cot^2(kl)) \quad \text{......................... (4)}$$

This is a periodic function which reaches a maximum value at frequencies where the airspace length is equal to an odd multiple of 1/4— wavelength and has a value of zero at frequencies where the airspace length is equal to an even multiple of 1/4— wavelength. The maximum value of the absorption coefficient is given by

$$\alpha'_{max} = 4x/(1 + x)^2 \quad \text{....................................... (5)}$$

Comparison of equation 5 with equation 1 shows that, for small values of x,

$$\alpha'_{max} \approx 4\alpha$$

The addition of the reflecting surface has increased the absorption coefficient by a factor of 4 at some frequencies, compared with the fabric in free space.

Table 1 gives values of α_{max} calculated from equation 5 for a range of fabric resistances. The limit of acceptability for low-resistance fabrics has been set at 100 rayls, indicated by the dotted line in Table 1. (See Table at end of text). It is evident that even acceptable, low-resistance fabrics can give fairly high values of absorption coefficient when used to cover a reflecting surface.

Although fabric covers are not normally applied over surfaces which are perfectly reflecting at all frequencies** some types of acoustic treat-

* "live" and "dead" are subjective acoustic terms used to describe areas with a long reverberation time or a short reverberation time respectively. They are relative, not absolute terms, i.e. a small 'live' area may have a shorter reverberation time than a larger 'dead' area.

** Heavy fabric drapes are occasionally used in front of and close to wall surfaces to provide mid-and-high-frequency sound-energy absorption. A calculation, based on heavy velvet drapes (≈200 rayls) spaced 150 mm from a wall gives a normal incidence absorption coefficient of approximately 0.9 at 570 hz.

ment are reflective at some frequencies. In particular, low-frequency absorbers usually have either completely unperforated or very sparsely perforated front covers which can reflect about 90% of the incident sound energy at high frequencies.

Equation 5 forms the basis of a method of measuring the true fabric acoustic resistance using a time-varying air-velocity. The normal method of measuring fabric resistance consists of passing a controlled, continuous flow of gas (air) through a sample of known cross-sectional area. The resulting pressure difference, measured by means of a sensitive manometer, is used to derive the flow-resistance. The two main instrumental drawbacks to this method are the requirement for a well-controlled and accurately measured airflow and, because the pressure difference generated can be small, a sensitive and therefore delicate manometer. A significant fundamental drawback is that the resulting measurement inevitably represents the air-flow resistance which, for some types of material is significantly different to the acoustic resistance. However, if a sample of fabric is placed in an impedance tube at a known distance, l, in front of a solid backing and the absorption coefficient measured at a frequency of $344/4l$ Hertz, then equation 5 can be used to find the fabric acoustic resistance. The measurement frequency can be changed by varying the spacing, l.

If the surface covered by the fabric is neither a perfect absorber nor a perfect reflector then equation 2 must be used. For most surfaces, the surface resistance factor, y, will be unknown. An approximate value can be obtained from the measured absorption coefficient of the uncovered surface, as in appendix 2.

3. Experimental Verification

3.1. Normal incidence measurements

Measurements were carried out using an impedance tube to determine the absorption coefficient of a fabric sample and solid (reflecting) backing with a 48 mm airspace between. Fig. 2 shows the results of these measurements, together with the theoretical absorption coefficient calculated from equation 4 for a fabric acoustic resistance of 0.15 $\rho_0 c$ (60 rayls). The fabric sample had a measured flow-resistance of about 40 rayls.

An airspace of 48 mm corresponds to a first maximum in the absorption coefficient characteristic at a frequency of approximately 1770 Hz. The measurements on the sample at 1770 Hz give a result for the absorption coefficient of 0.48, corresponding to an acoustic resistance of 0.162 $\rho_0 c$ (= 66 rayls). The measurements also confirm the region of low absorption coefficient around 3.5 kHz, although none of the measured values is as low as the corresponding calculated value, in this frequency region. With the apparatus available, it was not possible to make measurements at frequencies higher than about 4.2 kHz or lower than about 160 Hz.

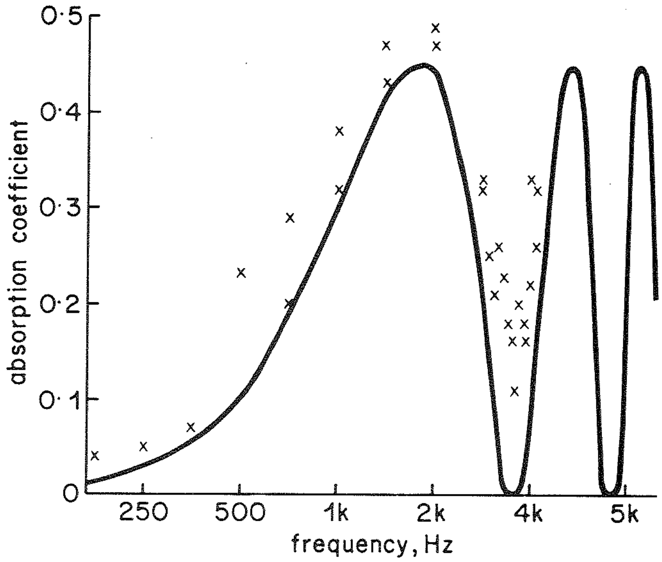

Fig. 2 — Normal incidence absorption coefficient of fabric layer, spaced at 48 mm in front of a reflecting surface.

—— Theoretical (x = 0.15)
× Measured

3.2. Random incidence measurements

3.2.1. Fabric covering over absorbing surface.

Comparative measurements of the effect of a fabric covering* were made using two different types of wide-band modular absorber. The first of these types was set of 24 A3 modules which consist of 20% open area, perforated hardboard over 30 mm mineral-wool and a 154 mm airspace. The effect of the perforated front-cover was such as to reduce the effective absorption coefficient progressively for frequencies above 4 kHz. Fig. 3 shows the absorption coefficients as functions of frequency, derived from reverberation room measurements, for both the fabric-covered and the uncovered absorbers. The spacing between the

* The fabric was from the approved range of lightweight fabrics and has a flow-resistance of about 40 rayls. It was currently in common use for this application.

fabric and the front surface of the absorbers was approximately 6 mm. At frequencies between about 630 Hz and 4 kHz, the uncovered treatment shows almost perfect absorption ($\alpha \approx 1.0$) which is reduced by a somewhat variable but probably significant amount by the fabric covering. This reduction in absorption coefficient averaged approximately 0.1 over this frequency range and was about 10 times the reduction predicted by the simple theory (Equation A2.13).

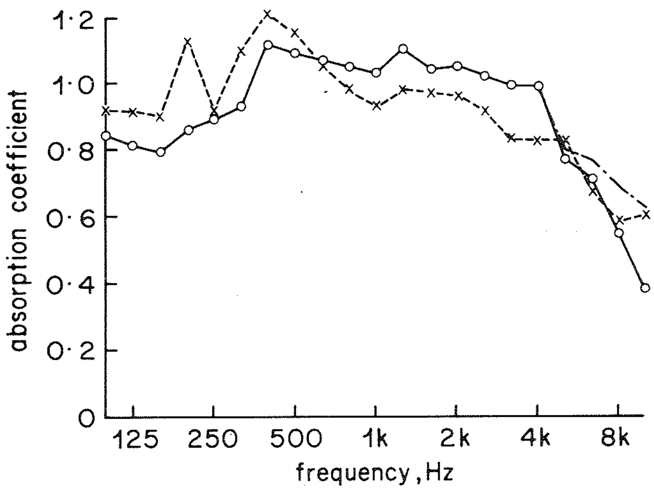

Fig. 3 — Absorption coefficient characteristic, A3 modular absorber

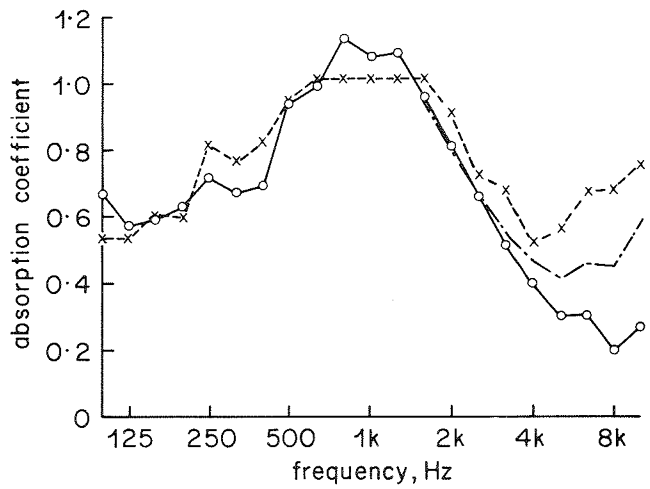

Fig. 4 — Absorption coefficient characteristic, A7 modular absorber

o——o without fabric cover x- - -x with fabric cover —·— calculated effect of fabric cover

At frequencies above about 4 kHz, the absorption coefficient of the treatment was reduced by the perforated front cover until at extreme high frequencies, it became about 0.4. Over most of this high-frequency region, the fabric cover has little effect on the absorption coefficient. Only at the highest frequency, 10 kHz, does the fabric-cover increase the measured absorption coefficient.

Fig. 3 also shows the calculated effect of the fabric cover as a function of frequency, based on the normal-incidence theory of equation 2. As this simple theory cannot consider absorption coefficients greater than unity the calculated results are not given for those frequencies at which the uncovered treatment has an absorption coefficient greater than unity.

The second type of wide-band treatment tested consisted of a set of ten special absorbers, intended for the treatment of a large music studio. The effect of the 20% perforated front cover on the high-frequency performance had been augmented by the addition of two layers of 0.05 mm thick polyester film. Fig. 4 shows the measured absorption coefficient characteristics of these absorbers both with and without the fabric covering. The range of frequencies over which uncovered treatment had an absorption coefficient near to unity was smaller than that for the A3 absorbers, extending from approximately 500 Hz to 1.6 kHz. Within this frequency range, the fabric covering caused a slight reduction in absorption coefficient, averaging approximately 0.03. At frequencies above 1.25 kHz, the uncovered treatment shows a pronounced fall in absorption coefficient as a consequence of the combined effects of the perforated front panel and the polyester film. Because the uncovered treatment reflects 70% or more of incident sound energy at frequencies above 4 kHz, the addition of the fabric covering causes a larger relative increase in absorption coefficient in this frequency range than it did for the A3 absorbers (Fig.3). the actual values of the absorption coefficients at 10 kHz for these two fabric covered treatments are very similar to each other. The effect of the polyester film has been partly nullified by the addition of the fabric; only over the frequency range 2.5 kHz to 5 kHz is the absorption coefficient of the polyester covered absorber significally lower than the standard A3, when both are covered with fabric. Fig. 4 also shows the calculated effect of the fabric cover in the same way as in Fig. 3.

3.2.2. Fabric covering over reflecting surface

In the range of frequencies where a lightweight fabric covering would be expected to

produce significant changes in absorption coefficient, that is, above about 1 kHz, a low-frequency absorber is a good reflector of sound energy. The usual types of low-frequency absorbers have either a perforated front cover with an open-area to total-area ratio of 0.5% or an unperforated front cover. Both of these finishes are good reflectors of high-frequency sound-energy. A measurement was carried out on a set of 24 A2 modular absorbers in the reverberation room both with and without the same lightweight fabric front covering used for the tests in section 3.1. Fig. 5 shows the absorption coefficient characteristics obtained. Over the frequency range 250 Hz to 1.6 kHz the addition of the fabric caused a slight but consistent increase in the absorption coefficient, averaging approximately 0.1. At frequencies of 2.0 kHz and above, the increase in absorption coefficient becomes

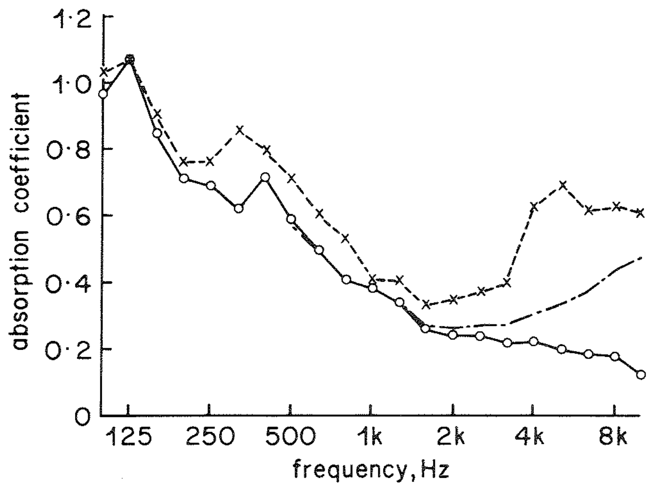

Fig. 5 — Absorption coefficient characteristic, A2 modular absorber

o——o without fabric cover

x– – –x with fabric cover

—·— calculated effect of fabric cover

progressively greater up to 5 kHz. This increase at high frequencies is similar to that measured with the polyester-covered high-frequency absorbers described in section 3.1. Fig. 5 also shows the calculated effect of the fabric covering, based on the normal-incidence theory of Appendix 2, for the same 6 mm airspace.

4. Discussion of results

Although the measurements carried out in the course of this work were made using modular absorbers, both the calculated and experimental effects of the fabric coverings apply generally and the results are relevant to the use of other types of acoustic treatment.

The normal incidence measurements, Section 3.1, show a fair degree of agreement with the theory, although the apparent acoustic resistance (66 rayls) is significally higher than the measured flow resistance ($\approx$ 40 rayls).

The results of the measurements carried out with random incidence sound waves, section 3.2, show that acoustic treatment which has a high absorption coefficient is affected very little by the addition of a fabric cover. The measured change in absorption coefficient was about ten times the theoretical change, but, as both are small, the effect is of little practical significance.

However, in the case of both reflecting surfaces and acoustic treatment at frequencies where the absorption coefficient is low, the changes in acoustic performance brought about by even a lightweight fabric cover can be considerable. Even approved lightweight fabrics can increase the absorption coefficient of a reflecting surface up to 0.6 or more. The theoretical analysis shows that, for small spacings between the fabric and the surface, the increase in absorption coefficient is restricted to a relatively narrow band of high frequencies. The measured results show that, in practice, the frequency range over which the increase takes place is wider than that shown by the theory, and that the magnitude of the effect is not less than is shown by the theoretical analysis. All of these factors combine to show that fabric coverings, even over moderately reflective surfaces, can introduce significant amounts of high-frequency sound-energy absorption. The frequency range over which this can occur is directly dependant on the spacing between the fabric cover and the reflective surface; it is significant within the frequency range normally considered in the acoustic design of studios and control rooms for spacings equal to or greater than about 6 mm.

This effect may be exploited by the intentional covering of hard, reflective surfaces with heavy fabric spaced at some distance; for example, by draping heavy curtains in front of a wall surface. By "gathering" the curtains, the effective airspace is made non-uniform and a consistently high value of absorption coefficient can be achieved over the whole of the mid and high frequency range.

5. Conclusions

This study of the acoustic effect of fabric coverings over acoustic treatment has shown that the effect on the absorption coefficient can be

significant for some types of treatment.

The acoustic performance of acoustic treatment which absorbs most of the sound energy incident on its surface is not significally affected by the addition of lightweight fabric covering. However, with surfaces which absorb less than about 30% of the incident sound energy (the remainder being reflected) the fraction of sound energy absorbed by a combination of the surface and a lightweight fabric cover may be much greater than for the surface alone. The increase is dependent on the reflectivity of the surface, the resistance of the fabric and on the airspace separating the two as well as the frequency of the sound. With practical methods of construction, the effect is only significant at high frequencies.

In the design of studios and control rooms with fabric-covered acoustic treatment, the effect of the fabric on the performance of the treatment must be considered in those cases where the fabric covers a surface which reflects a significant fraction of the sound energy at mid- and high-frequencies. To minimise the effect, within the frequency range usually considered, the airspace between the fabric and the surface should be as small as possible, preferably less than 6 mm.

Appendix 1
The calculation of absorption coefficients.

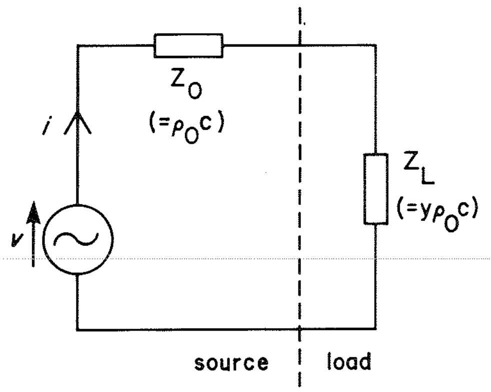

Fig. A1 — Impedance analogy of sound energy absorption.

The circuit of Fig. A1 is the impedance analogy representing a sound wave incident on the surface, where:-

Z_o ($= R_o + jX_o$) represents the acoustic radiation impedance of the surface.

Z_L ($= R_L + jX_L$) represents the acoustic impedance of the surface.

v represents the acoustic pressure.

i represents the acoustic volume velocity.

Considering Fig. A1

$$i = v/((R_o + R_L)^2 + (X_o + X_L)^2)^{1/2} \quad \text{.......... A1.1}$$

Power absorbed by the surface, P_a, is given by

$$P_a = \frac{R_L.v^2}{((R_o + R_L)^2 + (X_o + X_L)^2)} \quad \text{.............. A1.2}$$

If the surface is replaced by a surface giving the maximum power absorption, P_{ref}

i.e. $Z_L = R_o - jX_o$ A1.3

then $P_{ref} = v^2/4R_o$ A1.4

The absorption coefficient, α, is given by

$$\alpha = \frac{P_a}{P_{ref}} = \frac{4R_L R_o}{((R_o + R_L)^2 + (X_o + X_L)^2)} \quad \text{.......... A1.5}$$

In the case of a unit area of surface with a normally incident plane wave, Z_o can be replaced by the free space acoustic resistance, $\rho_o c$, where ρ_o is the density of the medium and c the velocity of sound propagation in the medium. Also, for the purpose of this work, surface impedance, Z_L, can be replaced by the resistance, $y.\rho_o c$, where y is the surface resistance factor. In the frequency range where fabric coverings have a significant effect and for the types of surfaces considered, the surface reactance component, X_L, is small. Hence equation A1.5 can be simplified to

$$\alpha = 4y/(1 + y)^2 \quad \text{.................................... A1.6}$$

Appendix 2
The absorption coefficient of fabric covered acoustic treatment

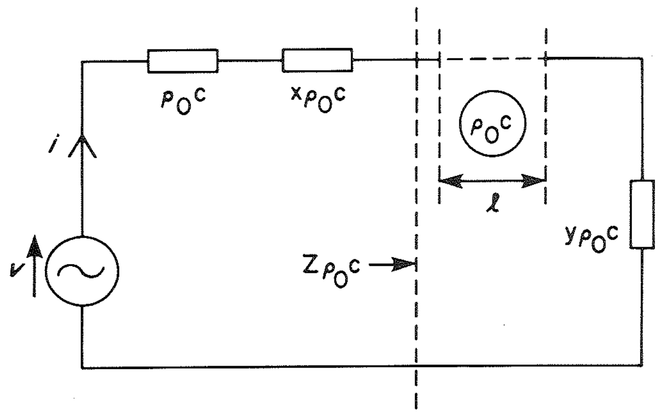

Fig. A2 – Impedance analogy of sound energy absorption, with spaced fabric layer.

Fig. A2 shows the impedance analogy representing a plane wave incident on a unit area of acoustic treatment, as in Fig. A1, with the addition of a spaced, thin fabric cover at a distance l where,

ρ_o	=	standard air density
c	=	velocity of sound, standard conditions
$\rho_o c$	=	plane wave, free-space acoustic impedance and the radiation impedance of unit area
$y\rho_o c$	=	acoustic resistance of the acoustic treatment
$x\rho_o c$	=	acoustic resistance of the fabric cover
l	=	length of the transmission line representing the airspace between fabric cover and acoustic treatment
v	=	impedance analogy of acoustic pressure
i	=	impedance analogy of acoustic volume velocity

All the above impedances are real. The assumption has been made that the acoustic treatment has no significant reactive components. This is a valid assumption for typical absorber constructions within the frequency range considered. It excludes the consideration of absorbing systems consisting wholly or partly of vibrating panels or air-masses. However, such resonant systems are not usually encountered within the frequency range where fabric covers have a significant effect.

<u>To calculated the impedance factor y, of the uncovered acoustic treatment.</u>

As this work is concerned only with the modifying effects of fabric covers over acoustic treatment, and, for the types of acoustic treatment being considered, the reflection and absorption coefficients are reasonably independent of the angle of incidence, it is justifiable to calculate the resistance factor, y, from the measured performance of the uncovered acoustic treatment. Such a procedure would not normally be justifiable as the analogy of Fig. A1 represents a plane wave normally incident on the acoustic treatment, whereas the measurement is carried out in the reverberation room with sound waves incident at all angles randomly.

From Fig. A1, the absorption coefficient, α, can be derived, as in appendix 1.

Rearranging equation A1.6 gives:

$$y = (2 - \alpha \pm 2\sqrt{1-\alpha})/\alpha \quad \text{A2.1}$$

Thus, except for α=1, there are two possible values of y for any value of α. For the given conditions, the smaller of these two values ($y<1$) is not physically realizable, therefore equation A2.1 can be simplified to

$$y = (2 - \alpha + 2\sqrt{1-\alpha})/\alpha \quad \text{A2.2}$$

<u>To calculate the effect of the fabric covering</u>

If a fabric cover is added to the absorber represented by Fig. A1, the resulting combination can be represented by the analogy of Fig. A2. The length of the transmission line, of characteristic impedance $\rho_o c$, representing the airspace between the fabric and the front surface of the absorber is given by l. The input impedance, Z_{in}, of a transmission line of length l, characteristic impedance Z_o, terminated by an impedance Z_L at a frequency f is given by :

$$Z_{in} = \frac{(Z_L \cos(2\pi fl/c) + j.\, Z_o \sin(2\pi fl/c))}{(j.\, Z_L \sin(2\pi fl/c) + Z_o \cos(2\pi fl/c).)}$$

Substituting the variables of Fig. A2. gives

$$Z\rho_o c = \frac{\rho_o c\,(y \cot(kl) + j.\,1)}{(\cot(kl) + j.\,y)} \quad \text{A2.3}$$

where $k = 2\pi f/c$

$$\therefore \quad Z = \frac{y(\cot^2 (kl) + 1) + j.\cot (kl).(1 - y^2)}{(\cot^2 (kl) + y^2)} \quad \text{A2.4}$$

or $Z = Z_R + j.Z_I$ A2.5

where $$Z_R = \frac{y(\cot^2 (kl) + 1)}{(\cot^2 (kl) + y^2)} \quad \text{A2.6}$$

and $$Z_I = \frac{(1 - y^2) \cot (kl)}{(\cot^2 (kl) + y^2)} \quad \text{A2.7}$$

where y is defined by equation A2.2.

For Fig. A2, the total circuit series impedance Z_T, is given by

$$Z_T = \rho_o c\,((Z_R + x + 1) + j.Z_I) \quad \text{A2.8}$$

$$\therefore \quad i = v/\rho_o c((Z_R + x + 1) + j.Z_I)) \quad \text{A2.9}$$

$$|i| = v/(\rho_o c((Z_R + x + 1)^2 + Z_I^2)^{1/2}) \quad \text{A2.10}$$

Power absorbed, P_a is given by

$$P_a = \frac{v^2 (Z_R + x)}{(\rho_o c((Z_R + x + 1)^2 + Z_I^2))} \quad \text{A2.11}$$

Absorption coefficient with fabric covering, α', is given by $\alpha' = P_a/P_{ref}$

$$\therefore \quad \alpha' = 4(Z_R + x)/((Z_R + x + 1)^2 + Z_I^2) \quad \text{A2.12}$$

taking $P_{ref} = v^2/4\,\rho_o c$

and Z_R and Z_I are defined by equations A2.6 and A2.7.

Equation A2.12 can be simplified for two extreme cases.

If the uncovered absorber is near perfect, that is, as $\alpha \to 1$ then: $y \to 1$

$$\text{and } \alpha' \to 4(1 + x)/(2 + x)^2 \quad \text{A2.13}$$

For all positive, non-zero values of x (real fabric acoustic resistance), $\alpha' < \alpha$, that is, the fabric-covered absorber will absorb less sound energy than the uncovered absorber.

If the uncovered absorber is a perfect reflector, that is, $\alpha = o$, then:

$y = \infty$

$$\text{and } \alpha' = 4x/((1 + x)^2 + \cot^2 (kl)) \quad \text{A2.14}$$

This is a periodic function which reaches a maximum value at frequencies where the airspace length is an odd multiple of 1/4 —wavelength. It has a value of zero at frequencies where the airspace length is an even multiple of 1/4—wavelength.

The maximum value of the absorption coefficient is given by

$$\alpha'_{max} = 4x/(1 + x)^2 \quad \text{A2.15}$$

Table 1

x	α' max	fabric resistance (mks rayls)	
0.05	0.181	20	approved
0.10	0.331	40	approved
0.2	0.556	80	approved
0.3	0.710	120	non-approved
0.4	0.816	160	non-approved
0.5	0.889	200	non-approved